3D elastic wave modeling using the complex screen method

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Summary

Fast modeling methods in complex model are crucial for the applications of seismic methods including forward modeling, imaging and inversion. In this paper, with an iterative algorithm, the complex screen method based on one-way wave equation is used to calculate both forward propagation and primary reflection. The method, which uses a small angle approximation, has a high efficiency. Synthetic seismograms for two and three dimensional elastic models are calculated with this method and compared with that generated by finite-difference methods. The results show that for small and medium scattering angles, this method gives reasonably good results.

Introduction

Fast modeling methods and algorithms in complex heterogeneous media, especially for 3-D media, are crucial to the applications of seismic methods in complex structure including the development of interpretation, imaging and inversion methods. Finite-difference and finite-element algorithms are very flexible. Theoretically, they can be applied to arbitrarily heterogeneous medium. However, they are very time consuming. High-frequency asymptotic methods, such as ray based methods, provide high computation efficiency for smooth 3D models. However, they fail in dealing with complicated 3D volume heterogeneities. With these methods the frequency-dependent and wave related phenomena in complex media can not be correctly modeled. Born scattering formulation, ray-Born, or generalized Born scattering methods can model small volume complex heterogeneities in smooth background. However, they are not capable of modeling long distance propagation in complex media. It is necessary to develop intermediate modeling methods functioning between the full wave equation methods and the high-frequency asymptotic methods. The screen methods have been used to calculate the one-way forward propagations for both acoustic and elastic wave problems (e.g. Martin and Flatté 1988, Wu and Xie 1993 and Wu 1994), and used as back propagator for seismic wave migration in either acoustic or elastic media (e.g. Stoffa et al. 1990, Wu and Xie 1994). The screen methods are based on one-way wave equation that neglect backscattered waves, but correctly handle all the forward multiple-scattering effects, e.g., focusing/defocusing, diffraction, interference, and conversion between different wave types. For media where the resonance scattering or reverberations between heterogeneities can be neglected, the reflections will be dominated by single back scatterings. In this case, the screen method can also be adopted to calculate reflections. Xie and Wu (1995) tested the screen approximation for modeling elastic wave reflections. Wu, Huang and Xie (1995), Wu and Huang (1995) tested the method for acoustic reflections. Wu (1995) discussed various approximations for forward and backward scatterings of different wave types.

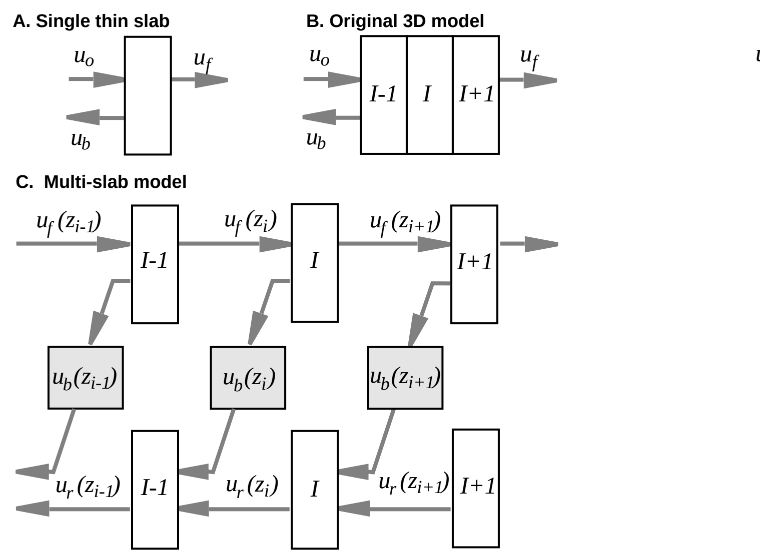

Figure 1: Sketch of the multiscreen method. Details pleas see the text.

The screen method has two main advantages. First, it neglects the multiple reflections so that the full wave equation can be replaced by the one-way wave equation, which considerably reduced the CPU time. Second, the screen method manipulates in successive 2D planes instead of the original 3D model, which tremendously reduced the demand for computer memories. These advantages make the complex screen method a very attractive candidate in dealing with complicated 3D models. The trade off of this method is that the reverberations are omitted from the calculation. And, once the small angle approximation is used, large angle scattering is less accurate. In exploration seismology, the reverberations are often omitted or eliminated through the primary data processing, which makes the lack of reverberation is tolerable for many practical applications. In this paper, under a small angle approximation, the complex screen method is used to calculate the synthetic seismograms for some 2D and 3D models. The results are compared with that from two and three dimensional elastic finite-difference methods. These results are generally consistent and which suggest that the complex screen method can be used for modeling responses from complicated elastic structures.

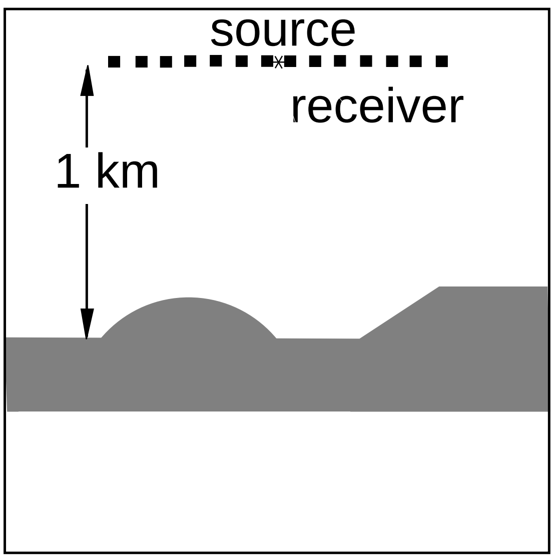

Figure 2: Two dimensional models used to compare the results from screen-approximation method and finite-difference method. The model is a 2D slice from the French model (French 1974). The parameters for the background medium is $V_P = 3.6 km/s$, $V_S = 2.08 km/s$ and $\rho = 2.2 gram/cm^3$. The intermediate layer has a -20% perturbation for both P- and S-wave velocities.

Brief description of the method

First, we consider waves incident on an inhomogeneous thin slab with thickness Δz and bounded between z_0 and z_1. As shown in Figure 1a, the incident wave $\mathbf{u}_0$ will generate backscattered wave $\mathbf{u}_b$ and forward propagated wave $\mathbf{u}_f$. At z_1, the exit side of the slab, the forward propagated field composed of primary wave and forward scattered P- and S-waves. It can be represented as the superposition of plane P- and S-waves

$$\mathbf{u}_f(\mathbf{x}_T, z_1) = \frac{1}{4\pi^2}\int d\mathbf{K}'_T \left[\mathbf{u}_f^P(\mathbf{K}'_T, z_1) + \mathbf{u}_f^S(\mathbf{K}'_T, z_1)\right] e^{i\mathbf{K}'_T\cdot\mathbf{x}_T} \tag{1}$$

where $\mathbf{K}'_T$ is the transverse wavenumber of plane waves, $\mathbf{x} = \mathbf{x}_T + z\mathbf{e_z}$ is the position, superscripts P and S denote P- and S-waves, and

$$\mathbf{u}_f^P(\mathbf{K}'_T, z_1) = e^{i\gamma'_\alpha \Delta z}\left[\mathbf{u}_0^P(\mathbf{K}'_T, z_0) + \mathbf{U}_f^{PP}(\mathbf{K}'_T, z_0) + \mathbf{U}_f^{SP}(\mathbf{K}'_T, z_0)\right] \tag{2}$$

$$\mathbf{u}_f^S(\mathbf{K}'_T, z_1) = e^{i\gamma'_\beta \Delta z}\left[\mathbf{u}_0^S(\mathbf{K}'_T, z_0) + \mathbf{U}_f^{SS}(\mathbf{K}'_T, z_0) + \mathbf{U}_f^{PS}(\mathbf{K}'_T, z_0)\right] \tag{3}$$

where $k_\alpha = \omega/\alpha$ and $k_\beta = \omega/\beta$ are P and S wavenumbers, $\gamma'_\alpha = \sqrt{k_\alpha^2 - {\mathbf{K}'_T}^2}$ and $\gamma'_\beta = \sqrt{k_\beta^2 - {\mathbf{K}'_T}^2}$ are longitudinal components of these wavenumbers. Phase advance operators $e^{i\gamma'_\alpha \Delta z}$ and $e^{i\gamma'_\beta \Delta z}$ propagate the incident and scattered fields from z_0 to z_1. The reflected wave is composed of reflected P- and S-waves. At z_0, the reflected wave can be expressed as

$$\mathbf{u}_b(\mathbf{x}_T, z_0) = \frac{1}{4\pi^2}\int d\mathbf{K}'_T \left[\mathbf{u}_b^P(\mathbf{K}'_T, z_0) + \mathbf{u}_b^S(\mathbf{K}'_T, z_0)\right] e^{i\mathbf{K}'_T\cdot\mathbf{x}_T} \tag{4}$$

where

$$\mathbf{u}_b^P(\mathbf{K}'_T, z_0) = \mathbf{U}_b^{PP}(\mathbf{K}'_T, z_0) + \mathbf{U}_b^{SP}(\mathbf{K}'_T, z_0) \tag{5}$$

$$\mathbf{u}_b^S(\mathbf{K}'_T, z_0) = \mathbf{U}_b^{SS}(\mathbf{K}'_T, z_0) + \mathbf{U}_b^{PS}(\mathbf{K}'_T, z_0) \tag{6}$$

In the above equations, $\mathbf{U}$ denotes scattered waves. The subscripts f and b denote forward and backward scatterings, respectively. Superscripts PP, PS, SP and SS indicate the scattering between different wave types. Detailed expressions of these scattered fields can be found in Xie and Wu (1995). Figure 1 shows how to calculate the interaction between an incident wave and a 3D heterogeneous model with an iterative method. First, the 3D model is divided into a series of thin slabs as shown in Figure 1b. The Ith slab is between z_i and z_{i+1}. The equations (1) through (6) provide formulas for calculating the interaction between incident wave and a single thin slab. With these equations, from the incident wave $u_f(z_i)$, we can calculate the transmitted field $u_f(z_{i+1})$ and backscattered field $u_b(z_i)$. The transmitted field is used as the input for the next slab and in this way the forward propagated field in the entire model can be obtained. The backscattered field from the Ith slab is backuped temporarily. After finished the forward propagation, the backscattered fields are recollected, and once again the one-way propagator is used to propagate the back scattered field through the entire model to form the total reflection $u_r(z)$. In this way, all the multiple forward scatterings and single backward scatterings can be correctly handled.

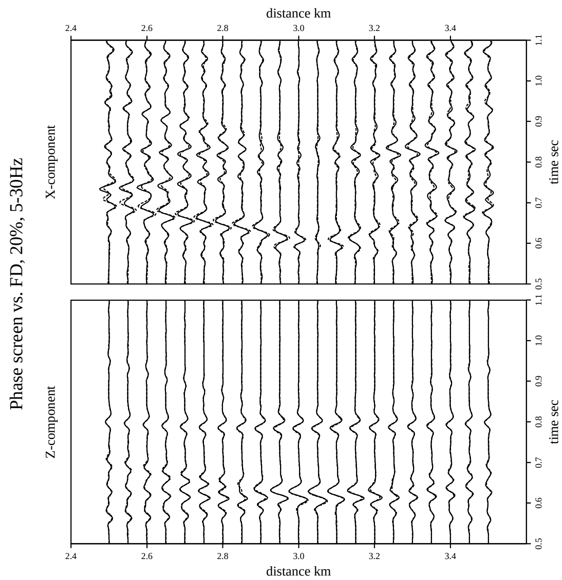

Figure 3: Comparison between results of 2D model from different methods. The solid lines are from screen method and the dash lines are from finite-difference method. The results show general consistency in both amplitude and arrival times.

Numerical examples

In this section we will give some numerical simulations to show the accuracy of this method. The first model is a 2D model which is a slice cut from the French model (French 1974). Figure 2 shows the velocity structure of this model. The parameters of the background medium are $V_P = 3.6km/sec$, $V_S = 2.08km/sec$ and $\rho = 2.2gram/cm^3$. The intermediate layer has a -20% perturbation for both P- and S-wave velocities. The source and the receivers are located $1km$ above the upper interface. The synthetic seismograms are calculated using the elastic complex screen method and 2D finite-difference method (Xie and Yao 1988). The free surface effects and primary arrivals have been properly removed from these results. The synthetic seismograms are basically reflections from the structure. The results are compared in Figure 3. Solid lines are from complex screen method and the dash lines are for that from finite-difference method. For Z-component, there are mainly P-wave energy. The energy arrived between 0.5 and 0.7 second are P to P reflections from different parts of the upper interface. Since the interface is rather complicated, there are several arrivals can be identified. The second group of energy are relatively simple. They are P to P reflections from the lower plane interface. The X-component of the synthetics is composed of P to S reflections from both interfaces as well as some P-wave energy. Generally speaking, the consistency between the two methods is very good.

The second model is 3D French model. The source-receiver configuration and velocity structure are similar to that shown in Figure 2 for 2D model except the perturbation for P- and S-wave velocities in the intermediate layer are both -10 %. The finite-difference result is generated from a nCUBE parallel machine by using 256 nodes (Cheng et al, 1994), and the screen result is generated by using a SUN workstation. Figure 4 shows the comparison between the different methods. Although these results are generally consistent, there are errors in both amplitude and phase for wide angle reflections. For 2D model the dip angle of the fault is about 30 degrees, while for 3D model the dip angle is about 40 degrees. The obliquely reflected waves have incident angle up to 45 degrees relative to the vertical direction. These results roughly indicate the limit of small angle approximation.

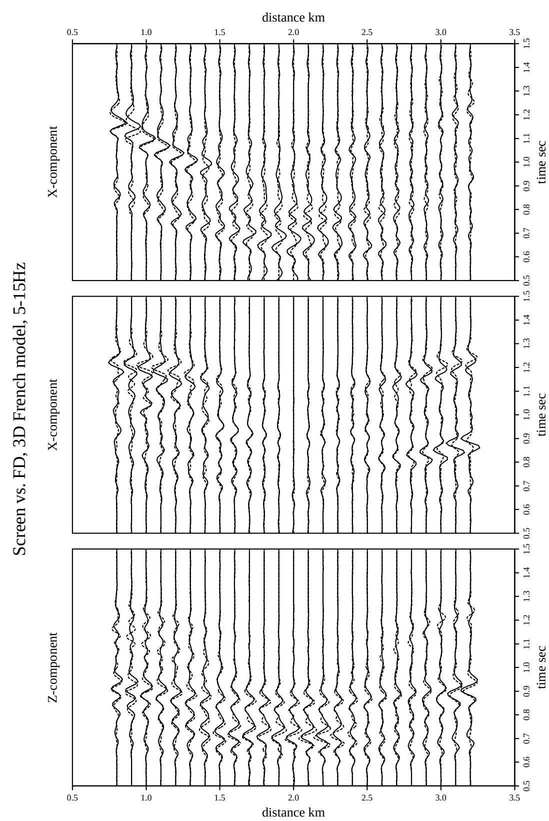

Figure 4: comparison between results of 3D model from different methods.

Conclusions

The elastic complex screen method is based on the one-way wave equation and small angle approximation. It provides an efficient way for propagating the wave field in both forward and backward directions. Using an iterative algorithm, the current method can calculate both forward propagating waves and primary reflections for a complicated model. Synthetic seismograms for 2D and 3D elastic models are generated with this method. The results are compared with that from the finite-difference method. For small to medium scattering angles, the method show good consistency with the finite-difference method. For wide angles, there are errors for both the phase and amplitude and suggest that the wide angle approximation is required.

Acknowledgements

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